BOSWELL-BÈTA

James Boswell Exam

VWO Mathematics C

Solution Key

Date:	Practice exam 2
Time:	3 hours
Number of questions:	6
Number of subquestions:	23
Number of supplements:	0
Total score:	67

Subject-specific marking rules and guidelines

1. For each error or mistake in calculation a single point will be subtracted from the maximum score that can be obtained for that particular part of the question.
2. If a required explanation, deduction or calculation has been omitted or has been stated incorrectly 0 points will be awarded, unless otherwise stated in the solution key. This is also the case for answers obtained by the use of a graphic calculator. Answers obtained by the graphic calculator should indicate how the graphic calculator has been used to obtain the answer. Candidates must make sure they mention formulas applied or provide lists and calculation methods used in their answers.
3. If a notational error has been made, but the error can be seen to have no influence on the final result, no points will be deducted from the total score. If, however, it is not possible to determine that there is no influence on the final result a point will be deducted from the final score.
4. A particular mistake in the answer to a particular exam question will lead to a deduction of points only once, unless the question is substantially simplified by the mistake and/or when the solution key specifies otherwise.
5. A repeated mistake made in the answer to different exam questions will lead to a deduction of points each time such a mistake has been made, unless the solution key specifies otherwise.
6. If only one example, reason, explication, explanation or any other type of answer is required and more than one has been given, only the first answer given will be graded. If more than one example, reason, explication, explanation or any other type of answer is required, only the first answers are graded, up to and including the number of answers specified by the exam question.
7. If the candidate fails to give a required unit in the answer to a question a single point will be subtracted from the total score, unless the unit has been specified in the exam question.
8. If during intermediate steps results are rounded, resulting in an answer different from one in which non-rounded intermediate results are used, one point will be subtracted from the total score. Rounded intermediate results may, however, be noted down
Exceptions to this rule are those cases in which the context of the question requires the rounding of intermediate results. The <u>maximum</u> number of points deducted from the total score due to rounding errors is 2 for the entire exam.

Examples for the exceptions to rule 8.
Rounding off intermediate results can be forced by the context if, for example
- The amount of money for a single good has to be rounded to two decimals;
- The number of persons, things, etc. in a concrete situation (i.e. not an average or expected value) has to be rounded to the nearest integer.

A required degree of accuracy can be forced by the context if, for example
- The answer would not be distinguishable from a trivial answer. This can occur with the rounding of growth factors or probabilities to 0 or 1. A probability of $\left(\frac{1}{6}\right)^5$ may be rounded to 0.0001 but not to 0.000.

The forced rounding up or down of answers can occur, for example
- If the exam question specifies a minimum or maximum amount. (For example, if the question is: 'What is the minimal distance an athlete has to jump to gain a certain number of points in a contest?')

The above examples by no means exhaust all possible cases.

Question 1: BMI and BSI

a	$l = 1.70$ gives $B = \dfrac{m}{1.70^2}$	1
	Insight that the equations $\dfrac{m}{1.70^2} = 18.5$ and $\dfrac{m}{1.70^2} = 25$ have to be solved	1
	Describing how these equations can be solved • algebraically: ○ $\frac{m}{1.70^2} = 18.5$ ○ $m = 18.5 \cdot 1.70^2$ $(= 53.465)$ • graphic-numerically: ○ $Y_1 = \frac{x}{1.70^2}$ and $Y_2 = 18.5$ ○ Option intersect gives $x = 53.465$ • algebraically: ○ $\frac{m}{1.70^2} = 25$ ○ $m = 25 \cdot 1.70^2$ $(= 72.25)$ • graphic-numerically: ○ $(Y_1 = \frac{x}{1.70^2}$ and$)$ $Y_2 = 25$ ○ Option intersect gives $x = 72.25$	1
	The smallest weight is 54 (kg) and the largest weight is 72 (kg)	1
b	$w = 0.96, m = 87.5$ and $l = 1.80$ gives $S = 0.96 \cdot 87.5^{-\frac{2}{3}} \cdot 1.80^{\frac{5}{6}}$ (≈ 0.079)	1
	$(0.078 < 0.079 < 0.080)$ Eric's health risk is 'high'	1
d	$S = w \cdot m^{-\frac{2}{3}} \cdot l^{\frac{5}{6}}$ $= w \cdot \dfrac{1}{m^{\frac{2}{3}}} \cdot l^{\frac{5}{6}}$ $= w \cdot \dfrac{1}{\sqrt[3]{m^2}} \cdot \sqrt[6]{l^5} \left(= \dfrac{w \cdot \sqrt[6]{l^5}}{\sqrt[3]{m^2}}\right)$	1 2

Question 2: Braille

a	There are $\left(\binom{6}{1} =\right)$ 6 possible characters with one tactile dot	1
	There are $\binom{6}{2} = 15$ possible characters with two tactile dots	1
	So there are $(6 + 15 =)$ 21 possible characters with one or two tactile dots	1
	The candidate may also draw all possibilities	
b	*Method 1:*	
	With 6 dots there are $2^6 - 1 = 63$ characters	1
	With 8 dots there are $2^8 - 1 = 255$ characters	1
	So there are $255 - 63 = 192$ extra characters	1
	The candidate may also give $2^8 - 2^6$ $(= 256 - 64) = 192$ as an answer, but only if he/she explains that for the difference it does not matter whether you in- or exclude the character with no tactile dots. If this explanation is missing, award at most 2 score points to this subquestion.	
	Method 2:	
	With 6 dots there are $\binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6} = 63$ characters	1
	With 8 dots there are $\binom{8}{1} + \binom{8}{2} + \binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8} = 255$ characters	1
	So there are $255 - 63 = 192$ extra characters	1
c	$\frac{36 \cdot 10^6}{7.380 \cdot 10^9} \cdot 100\% \; (= 0.487 \ldots \%)$	1
	The answer: 0.49%	1
d	*Method 1:*	
	(The growth factor per 35 years equals $g_{35\text{ years}} = 3$) The growth factor per year equals $g_{\text{year}} = 3^{\frac{1}{35}} = 1.0318 \ldots$	2
	The answer: 3.2% (per year)	1
	Method 2:	
	Insight that the equation $g^{35} = 3$ (or $36 \cdot g^{35} = 108$) has to be solved	1
	Describing how this equation can be solved • algebraically: ○ $g^{35} = 3$ ○ $g = \sqrt[35]{3} = 1.0318 \ldots$ • graphic-numerically: ○ $Y_1 = x^{35}$ and $Y_2 = 3$ ○ Option intersect gives $x = 1.0318 \ldots$	1
	The answer: 3.2% (per year)	1

Question 3: Test

a	The Amir's claim, 'not chatting' and 'asking question' are both necessary to pass the test. In Bo's claim, one of the two is enough to pass to test (so, for example, you can chat as long as you ask questions).	2
b	Using an implication arrow	1
	$\neg P \Rightarrow C \vee \neg E \vee \neg Q$	2
c	$\text{P(both the correct answer)} = 0.8^2\ (= 0.64)$	1
	For each incorrect answer: $\text{P(both this incorrect answer)} = 0.1^2\ (= 0.01)$	1
	$\text{P(both the same answer)} = 0.8^2 + 0.1^2 + 0.1^2 (= 0.8^2 + 2 \cdot 0.1^2) = 0.66$	1
d	$\text{P(the same answer to all ten questions)} = 0.66^{10}\ (= 0.015\,6 \dots)$	1
	The answer: 0.016	1

Question 4: Sauna

a	Each minute, the temperature increases by $\frac{56.3-20}{45} = 0.806 \dots$ (°C)	1
	At 12:10 the temperature was equal to $20 + 10 \cdot 0.806 \dots \approx 28.1$ (°C) (or $56.3 - 35 \cdot 0.806 \dots \approx 28.1$ (°C))	2
b	$t = 1$ gives $S = 200 - 180 \cdot 0.741^1 = 66.62$ (°C)	1
	13:15 corresponds to $t = 1.25$ $t = 1.25$ gives $S = 200 - 180 \cdot 0.741^{1.25} = 76.24 \dots$ (°C)	1
	$\frac{74.24 \dots - 66.62}{66.62} \cdot 100\%\ (= 14.45 \dots)$	1
	The answer: 14.5%	1
c	$200 - 180 \cdot 0.741^t = S$ $-180 \cdot 0.741^t = S - 200$	1
	$0.741^t = -\frac{1}{180}(S - 200)$ (or an equivalent expression)	1
	$t = \log_{0.741}\left(-\frac{1}{180}(S - 200)\right)$ (or an equivalent expression)	1
	$S = 100$ gives $t = \log_{0.741}\left(-\frac{1}{180}(100 - 200)\right) = 1.960 \dots$	1
	$0.960 \dots \cdot 60 = 57.6 \dots$ so the temperature reaches 100° C at 13:57 (or 13:58)	1
d	The equilibrium equals $\frac{99.3 + 100.2}{2} = 99.75$ (°C)	1
	The amplitude equals $100.2 - 99.75 = 0.45$ (°C)	1

Question 5: Forest

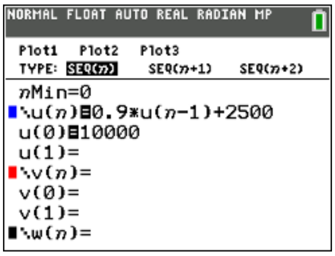
<table>
<tr><td>a</td><td>Each year 10% (or $\frac{1}{10}$) of the forest is cut down
(or each year 90% (or $\frac{9}{10}$) of the forest is not cut down)</td><td>1</td></tr>
<tr><td></td><td>Each year 2500 new trees are planted</td><td>1</td></tr>
<tr><td>b</td><td>Describing how T_3 can be calculated

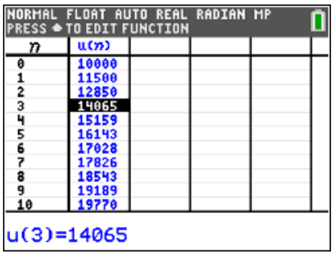
algebraically:

$T_1 \ (= 0.9 \cdot T_0 + 2500) = 0.9 \cdot 10\,000 + 2500 = 11\,500$
$T_2 \ (= 0.9 \cdot T_1 + 2500) = 0.9 \cdot 11\,500 + 2500 = 12\,850$
$T_3 \ (= 0.9 \cdot T_2 + 2500) = 0.9 \cdot 12\,850 + 2500 = 14\,065$

graphic-numerically:

$u(n) = 0.9 \cdot u(n-1) + 2500$ with $u(0) = 10\,000$
A table shows:

<table>
<tr><th>n</th><th>T_n</th></tr>
<tr><td>3</td><td>14 065</td></tr>
</table>




</td><td>2</td></tr>
<tr><td></td><td>The increase is $14\,065 - 10\,000 = 4065$ trees</td><td>1</td></tr>
<tr><td>c</td><td>X = number of pieces of land (in the sample) on which the plant species grows</td><td></td></tr>
<tr><td></td><td>Insight that $X \sim Bin\left(20, \frac{1}{2}\right)$</td><td>1</td></tr>
<tr><td></td><td>$\mathrm{P}(X \leq 7) = binomcdf\left(20, \frac{1}{2}, 7\right)$ $(= 0.131\,5 \ldots)$</td><td>1</td></tr>
<tr><td></td><td>The answer: 0.132</td><td>1</td></tr>
</table>

*see the next page for question **d**.*

<table>
<tr><td>d</td><td>Method 1:</td><td></td></tr>
<tr><td></td><td>$X =$ number of pieces of land (in the sample) on which the plant species grows</td><td></td></tr>
<tr><td></td><td>Insight that X is binomially distributed with $p = 0.05$ and unknown n (the number of pieces of land in the sample)</td><td>1</td></tr>
<tr><td></td><td>$\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0)$
$= 1 - binompdf(n, 0.05, 0)$

If the candidate uses a graphing calculator with which the probability $\mathrm{P}(X \geq 1)$ can be calculated directly, do not deduct points for not using the complement rule.</td><td>1</td></tr>
<tr><td></td><td>Describing how the inequality $1 - binompdf(n, 0.05, 0) > 0.90$ can be solved

$Y_1 = 1 - binompdf(x, 0.05, 0)$
A table shows:
<table>
<tr><th>n</th><th>$\mathrm{P}(X \geq 1)$</th></tr>
<tr><td>44</td><td>0.8953</td></tr>
<tr><td>45</td><td>0.9006</td></tr>
</table></td><td>1</td></tr>
<tr><td></td><td>The answer: at least 45 (pieces of land)</td><td>1</td></tr>
<tr><td></td><td>Method 2:</td><td></td></tr>
<tr><td></td><td>$\mathrm{P}(\text{at least one piece of land with the species}) = 1 - \mathrm{P}(\text{no piece of land with the species})$
$= 1 - 0.95^n$</td><td>2</td></tr>
<tr><td></td><td>Describing how the inequality $1 - 0.95^n > 0.90$ can be solved
algebraically:
$1 - 0.95^n = 0.90$
$0.95^n = 0.10$
$n = {}^{0.95}\log(0.10)$ $(= 44.8 \ldots)$
graphic-numerically:
$Y_1 = 1 - 0.95^x$
A table shows:
<table>
<tr><th>n</th><th>$\mathrm{P}(\text{at least one piece of land with the species})$</th></tr>
<tr><td>44</td><td>0.8953</td></tr>
<tr><td>45</td><td>0.9006</td></tr>
</table></td><td>1</td></tr>
<tr><td></td><td>The answer: at least 45 (pieces of land)</td><td>1</td></tr>
</table>

Question 6: Golf balls

a	$X =$ the weight of the manufacturer's golf balls (in grams) $X \sim Norm(45.5, 0.15)$	
	The maximum weight equals $1.62 \cdot 28.35 = 45.927$ (grams)	1
	$\mathrm{P}(X > 45.927) = normalcdf(45.927, 10^{99}, 45.5, 0.15)$ $(= 0.002\,20\,\ldots)$	1
	The answer: $(0.002\,20\,\ldots \cdot 100\% \approx)$ 0.22%	1
b	$Y =$ the diameter of the manufacturer's golf balls (in millimetres) $Y \sim Norm(43.25, 0.25)$	
	$invNorm(0.01, 43.25, 0.25)$ $(= 42.668\,\ldots)$	1
	The answer: 42.67 (millimetres)	1
c	*Method 1:*	
	Insight that the equation $-\frac{1}{900}(x-150)^2 + 25 = 0$ has to be solved	1

	Describing how this equation can be solved
	• algebraically:
	  ○ $\frac{1}{900}(x-150)^2 = 25$
	  ○ $(x-150)^2 = 22\,500$
	  ○ $x - 150 = 150$ or $x - 150 = -150$
	  ○ $x = 300$ $(\vee\ x = 0)$
	• graphic-numerically:
	  ○ $Y_1 = -\frac{1}{900}(x-150)^2 + 25$
	  ○ Option zero gives $x = 300$ (and $x = 0$)
		1

	Calculating whether the golf ball travels farther than 320 yards *Method 1:* $\frac{300}{0.9144} \approx 328$ yards, so the golf ball travels farther than 320 yards (because $328 > 320$) *Method 2:* $320 \cdot 0.9144 = 292.608$ metres, so the golf ball travels farther than 320 yards (because $300 > 292.608$)	1
	Method 2:	
	320 yards is equal to $320 \cdot 0.9144 = 292.608$ metres	1
	$h = -\frac{1}{900}(292{,}608 - 150)^2 + 25 \approx 2.4$ (metres) (or a different degree of accuracy)	1
	($2.4 > 0$ so) the golf ball is not yet on the ground. So the golf ball travels farther than 320 yards.	1

d	Expanding the brackets in $h = -\frac{1}{900}(x-150)^2 + 25$ $\left(= -\frac{1}{900}(x-150)(x-150)+25\right)$: $h = -\frac{1}{900}(x^2 - 150x - 150x + 22\,500) + 25$	1
	$h = -\frac{1}{900}(x^2 - 300x + 22\,500) + 25$ $h = -\frac{1}{900}x^2 + \frac{1}{3}x - 25 + 25$	1
	$h = -\frac{1}{900}x^2 + \frac{1}{3}x$ So $a = -\frac{1}{900}$ and $b = \frac{1}{3}$	1